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Error of quantum-logic simulation via vector-soliton collisions

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Abstract
In a concept of simulating the quantum logic with vector solitons by the author (Janutka 2006 *J. Phys. A: Math. Gen.* **39** 12505), the soliton polarization is thought of as a state vector of a system of cebits (classical counterparts of qubits) switched via collisions with other solitons. The advantage of this method of information processing compared to schemes using linear optics is the possibility of the determination of the information-register state in a single measurement. Minimization of the information-processing error for different optical realizations of the logical systems is studied in the framework of a quantum analysis of soliton fluctuations. The problem is considered with relevance to general difficulties of the quantum error-correction schemes for the classical analogies of the quantum-information processing.

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1. Introduction

Classical optical systems simulating the quantum logic are of interest because of high manipulating power enabling simple realization of logical gates [1, 2]. This simplicity is valuable in spite of disadvantages resulting from the classicality of the information that is transformed. The state of these systems is represented by a 2^n-component vector of complex numbers similar to the n-qubit state vector and any quantum-logic algorithm could be performed using such a classical system. However, the information encoded in this vector differs from genuine quantum information by the lack of the property of nonlocality since the classical counterparts of qubits (called cebits) cannot be identified with separate particles. The negative consequences of this fact are the exponential increase of resources necessary to simulate any logical operation with the number of cebits and uselessness of these systems for quantum communication purposes. It leads to elongation of the information-processing time compared to genuine quantum-information processing, thus limiting the size of the information register. Such quantum-logic simulators can be applied for testing non-trivial but small quantum circuits.

On the other hand, the quantum-logic simulation using linear optics demands perfect calibration of a system of many optical elements (beam splitters and phase shifters) [1]. Furthermore, the measurement of the state parameters (phases and amplitudes of the 2^n components of the single-photon state vector) demands many repetitions of single-photon transformations performed on consecutive indistinguishable photons. However, it is difficult to create a big number of indistinguishable photons. In [2], an alternative simulator of the quantum logic using nonlinear optics has been proposed. It uses a classical object—a multi-component vector soliton as a register of information instead of the quantum object—a single photon. The information is transformed there via collisions with other solitons instead of using passive optical elements (switching solitons of specific parameters—velocity and polarization simulate the action of quantum logical gates). The advantage of this method is simplicity of measurement of the register-state parameters compared to the measurement in the linear-optics scheme, which is described in detail in the following section.

In the present paper, in order to determine requirements necessary for ensuring the fault tolerance of this information-processing method, I study the time dependence of quantum fluctuations of the soliton parameters following the method of Haus and Lai [3, 4]. In particular, the evolution of the fluctuations of the polarization-vector components (the state vector) due to the soliton transmission and collisions is investigated. The transmission-induced fluctuations of modules of these polarization components are found to be independent of time while the fluctuations of their phases depend linearly on time essentially influencing the information-processing error. Values of these phase fluctuations determine efficiency of the information switching using the pulse collisions. It is found that the error induced by the register-pulse collisions with the switching solitons is not influenced by the time-dependent phase fluctuations.

A consequence of the lack of nonlocality of the information is inefficiency of usual quantum error-correcting networks. One cannot easily discard encoding cebits after the error correction, since they are not connected to separate particles as qubits are [5]. Furthermore, the error channels are not one-qubit channels (e.g. bit-flip or phase-flip channels) since a fluctuation of a single state-vector component contributes to error of many cebits. A simple method of the correction of the outcome-information error via repeating the algorithm (which would be inefficient for genuine quantum-computing systems) is proposed.

Following [2], two different realizations of the multi-component vector solitons are considered. These are the solitons propagated in the multi-component Bose–Einstein condensate (self-focusing (SF) described with multi-component nonlinear Schrödinger (NLS) equation) and in media displaying the self-induced transparency (SIT) of a V-bouquet configuration of [6].

Details of difficulties of optical realizations of the quantum-information processing and its simulation connected to the readout of the information-register state are described in section 2. They motivate studying the nonlinear realization of the quantum-logic simulator. In section 3, basics of vector solitons of the SF and SIT types are given including one-pulse solutions of their equations of motion as well as the transformation of the soliton polarizations under the collision of two pulses. In section 4, the time dependence of fluctuations of the soliton parameters and the collision-induced exchange of these fluctuations are analyzed. The concept of the quantum-logic simulation via collisions of vector soliton is outlined in section 5. The main source of information-processing errors and a method of their correction are discussed in section 6.

2. State readout of information register

In order to better motivate the interest in a vector-soliton-based logic, I compare consequences of the register-state readout method relevant to linear-optics- and to vector-soliton-based schemes of the quantum-logic simulation on the application complexity of both schemes.

2.1. Linear-optics quantum-logic simulator

The optical quantum-logic simulator proposed in [1] is a system of passive optical elements (beam splitters and phase shifters) creating an interferometer of 2^n income and 2^n outcome arms, where n is a number of cebits. A single photon passing through this processor represents an n-cebit state vector (of components $c_j \equiv |c_j|\,\mathrm{e}^{\mathrm{i}\varphi_j}$, $j = 1, 2, \ldots, 2^n$ each connected to one outcome arm of the interferometer) being an information register. The number of optical elements used in order to perform any logical operation of the quantum universal set (of: *CNOT*, *Hadamard*, $\pi/8$, *phase* gates [7]) with such a device grows exponentially with n.

Since one is unable to determine 2^n components of the outcome-state vector of the single photon via any measurement, one has to repeat the logical operations with many identical photons passing through the optical system. In order to measure the phase changes of the 2^n register-state parameters with relevance to a common control phase, we need to operate on a state vector of more than 2^n components. If our register state was a projection of a 2^{n+1}-component vector and 2^n control components $c \equiv |c|\,\mathrm{e}^{\mathrm{i}\varphi}$ were identical, we can measure the phases φ_j and the amplitudes $|c_j|$ of the register parameters via using 2^n interferometers consisting of single 50:50 beam splitters and counting photons at the outcome arms of the interferometers. Income arms of each of the interferometers are: one outcome arm of the logical subsystem (a register arm) and one outcome arm of the control subsystem (a control arm).

Let the numbers of photons counted at the outcome arms of the interferometer be denoted as n', n'_j and the number of photons in the income control arm denoted as n is known. The number of photons coming into the register arm $n_j = N|c_j|^2$ is to be determined as well as the quantum-phase difference of photons at both income arms $\varphi_j - \varphi$. Here, N denotes the whole number of photons passing through the system. The relation between the annihilation operators of photons in the income (a, a_j) and outcome (a', a'_j) arms

$$\begin{pmatrix} a' \\ a'_j \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\mathrm{i} \\ -\mathrm{i} & 1 \end{pmatrix} \begin{pmatrix} a \\ a_j \end{pmatrix} \equiv B \begin{pmatrix} a \\ a_j \end{pmatrix} \tag{1}$$

leads to

$$a'^{\dagger}_j a'_j - a'^{\dagger} a' = i\left(a^{\dagger} a_j + a^{\dagger}_j a\right). \tag{2}$$

Since $\left\langle a^{\dagger} a_j + a^{\dagger}_j a \right\rangle \propto \sin(\varphi_j - \varphi)$, one finds [8]

$$\sin(\varphi_j - \varphi) = \frac{1}{2} \frac{n'_j - n'}{\sqrt{n n_j}}. \tag{3}$$

Using $n + n_j = n' + n'_j$, the amplitude and phase of the state-vector parameter of the information register can be determined as follows:

$$\begin{aligned} |c_j| &= \sqrt{(n' + n'_j - n)/N}, \\ \varphi_j - \varphi &= \arcsin\left[\frac{1}{2} \frac{n'_j - n'}{\sqrt{n(n' + n'_j - n)}}\right]. \end{aligned} \tag{4}$$

Let us note that the necessity of using many photons in order to measure the state parameters is a source of inefficiency of linear-optics schemes of the quantum-information processing [9, 10]. However, even the implementation of the present linear-optics simulation of the quantum logic is connected to some difficulty resulting from consideration of the uncertainty of the phase-cosine operator

$$\begin{aligned}\left(\Delta_{a^\dagger a_j + a_j^\dagger a}\right)^2 &\equiv \frac{\langle A|(B^\dagger\otimes 1)\left(a^\dagger a_j + a_j^\dagger a\right)^2(B\otimes 1)|A\rangle - \langle A|(B^\dagger\otimes 1)\left(a^\dagger a_j + a_j^\dagger a\right)(B\otimes 1)|A\rangle^2}{\langle A|(B^\dagger\otimes 1)\left(a^\dagger a_j + a_j^\dagger a\right)(B\otimes 1)|A\rangle^2}\\ &= \frac{2N - N(N-1)(|c_j|^2 - |c|^2)^2 - N(cc_j^* + c_j c^*)^2}{N^2(cc_j^* + c_j c^*)^2}.\end{aligned} \tag{5}$$

Here $|A\rangle$ denotes the outcome state of the processor

$$|A\rangle \propto \underbrace{\left(a_1^\dagger + a_2^\dagger + \cdots + a_{2^n}^\dagger + a^\dagger + \cdots\right)}_{2^{n+1}}{}^N |0\rangle. \tag{6}$$

When the probabilities of photon observation at any outcome arm of the processor are similar ($|c_j|^2 = |c|^2 = 2^{-n-1}$), one estimates $\Delta_{a^\dagger a_j + a_j^\dagger a} \sim (2^n/N)^{1/2}$. Thus, the phase measurement demands much more repetitions of the algorithm than 2^n using consecutive indistinguishable photons. The indistinguishability of photons coming into the processor is important for avoiding decoherence which would be present when the optical beam was broadened. If the incoming beam was not perfectly coherent, its decoherence would increase after the beam passing through each optical element. The necessity of creating many indistinguishable photons is the reason of technical difficulties. Since these photons have to be created from a common source one after one, the effective information-processing time is much longer than the time of single-photon transmission.

2.2. *Quantum-logic simulation using vector solitons*

In the vector-soliton based scheme of the quantum-logic simulation [2], the n-cebit information register is represented by the 2^n-component vector of the soliton polarization $(c_1, c_2, \ldots, c_{2^n})$. The field-component envelopes of a multi-chromatic pulse $\epsilon_j(x,\tau)$ are equal to $c_j\epsilon(x,\tau)$ and the signal intensity of the j th component is defined as $w_j(x) \equiv \int_\tau^{\tau+T} |\epsilon_j(x,\tau')|^2\,\mathrm{d}\tau'$, where T is a time much longer than the pulse width divided by the soliton velocity.

Following the above measurement scheme relevant to the register-state readout for the linear-optics quantum-logic simulator, we consider data transmission with a 2^{n+1}-component pulse. It's polarization vector contains 2^n identical control components c. Performing the transformation of two polarization components

$$\begin{pmatrix} c' \\ c_j' \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\mathrm{i} \\ -\mathrm{i} & 1 \end{pmatrix} \begin{pmatrix} c \\ c_j \end{pmatrix} \tag{7}$$

via a collision with another (switching) soliton (following the method described in section 5) and measuring the outcoming-signal intensities $w'_{(j)}(x) \equiv \int_\tau^{\tau+T} |c'_{(j)}|^2|\epsilon(x,\tau')|^2\,\mathrm{d}\tau'$, one determines the polarization-component amplitudes and phases. Using the equality $|w| + |w_j| = |w'| + |w_j'|$, one arrives at

$$\begin{aligned}|c_j| &= \sqrt{(w' + w_j' - w)/W},\\ \varphi_j - \varphi &= \arcsin\left[\frac{1}{2}\frac{w_j' - w'}{\sqrt{w(w' + w_j' - w)}}\right],\end{aligned} \tag{8}$$

where $W(x) \equiv \int_{\tau}^{\tau+T} |\epsilon(x,\tau')|^2\, \mathrm{d}\tau'$ is the intensity of the multi-component pulse as a whole. Since the pulse-component intensities are classical observables, they can be determined at a single act of measurement. This fact makes vector solitons an attractive alternative of the single photons for using them as multi-cebit information registers.

3. Vector-soliton propagation and collisions

I outline realizations of the multi-component vector solitons in SF media and in media displaying SIT studying the pulse equations of motion and their one-soliton solutions. A general polarization transform performed via the soliton collision is written.

3.1. One-soliton solution of NLS equation

The envelope of an electromagnetic wave propagating in a nonlinear (Kerr) medium is described with the two-component NLS equation. It was solved by Manakov [11] within the inverse-scattering approach [12]. The propagation of two-component vector solitons predicted by him has been observed in different optical systems [13–15], and they were considered as the (classical) information bits switched via collisions with other Manakov solitons [16, 17]. Recently, many-component vector solitons (of the Manakov type) are observed in the multi-component atomic Bose–Einstein condensates [18]. These are matter waves described with multi-component Gross–Pitaevskii equation which is equivalent to the multi-component NLS equation [19, 20].

Considering the propagation of a multi-chromatic pulse, one uses the N-component NLS equation

$$\mathrm{i}\epsilon_{j,\tau} + \epsilon_{j,xx} + \frac{1}{2}\sum_{k=1}^{N} |\epsilon_k|^2 \epsilon_j = 0, \tag{9}$$

that is the equation of motion of the pulse-component envelopes. Here, $A_{,\tau} \equiv \partial A/\partial\tau$, $A_{,x} \equiv \partial A/\partial x$ denote differentials over renormalized time and position variables. The one-soliton solution of (9) takes the form

$$\epsilon_j(x,\tau) = 4\mathrm{i}c_j\zeta'' \exp[\mathrm{i}2\zeta' x + \mathrm{i}4(\zeta'^2 - \zeta''^2)\tau]\,\mathrm{sech}[2\zeta''(x - x_0) + 8\zeta'\zeta''\tau], \tag{10}$$

where $c_j \equiv |c_j|\,\mathrm{e}^{\mathrm{i}\varphi_j}$ denote components of a polarization vector of the unit length. The constant ζ ($\zeta' \equiv \mathrm{Re}\,\zeta$, $\zeta'' \equiv \mathrm{Im}\,\zeta$) is called a (complex) wavenumber.

3.2. One-soliton solution of SIT equations

Let us consider an $(N+1)$-level atomic medium coupled to an N-component optical pulse in such a way that the lowest atomic level is linearly coupled to others (a V-bouquet configuration of [6]) [21]. The pulse propagation is described with the Maxwell–Bloch equations and the slowly-varying envelope approximation is used for the decomposition of the Maxwell wave equation (see, e.g., [22]). The electronic-level occupation amplitudes b, a_j (state-vector components) for the ground and excited levels, respectively, satisfy $\sum_{j=1}^{N} |a_j|^2 + |b|^2 = 1$. A spectral distribution $g(\alpha)$ characterizing the inhomogeneous broadening of the medium is normalized to unity, $\int_{-\infty}^{\infty} g(\alpha)\,\mathrm{d}\alpha = 1$. Following [23, 24], the evolution of the density-matrix components $\lambda_{ij} \equiv 2a_i a_j^*$, $\lambda_{j0} \equiv 2a_j b^*$, $\lambda_{00} \equiv 2bb^*$ and of the envelopes of the pulse components ϵ_j is described with

$$\epsilon_{j,x} = \langle\lambda_{j0}\rangle, \tag{11a}$$

$$\lambda_{j0,\tau} + 2\mathrm{i}\zeta'\lambda_{j0} = \frac{1}{2}\epsilon_j\lambda_{00} - \frac{1}{2}\sum_{k=1}^{N}\epsilon_k\lambda_{jk}, \tag{11b}$$

$$\lambda_{ij,\tau} = \frac{1}{2}\epsilon_i\lambda_{j0}^* + \frac{1}{2}\epsilon_j^*\lambda_{i0}, \tag{11c}$$

$$\lambda_{00,\tau} = -\frac{1}{2}\sum_{k=1}^{N}\epsilon_k\lambda_{k0}^* - \frac{1}{2}\sum_{k=1}^{N}\epsilon_k^*\lambda_{k0}. \tag{11d}$$

Here $\langle A\rangle = \int_{-\infty}^{\infty} A(\alpha)g(\alpha)\,\mathrm{d}\alpha$, and a frequency detuning ζ' is assumed to be the same for all the coupled electromagnetic modes. Equations (11*b*)–(11*d*) are equivalent to

$$a_{j,\tau} + \mathrm{i}\zeta' a_j = \frac{1}{2}\epsilon_j b, \qquad b_{,\tau} - \mathrm{i}\zeta' b = -\frac{1}{2}\sum_{k=1}^{N}\epsilon_k^* a_k. \tag{12}$$

Solving the system of (11*a*) and (12) with the inverse scattering method, one finds the one-soliton scattering potentials

$$\epsilon_j(x,\tau) = 4c_j\zeta''\exp\{\mathrm{i}\omega'(\zeta)x + \mathrm{i}2\zeta'\tau\}\,\mathrm{sech}\{2\zeta''[\tau - \tau_0 + x/v(\zeta)]\} \tag{13}$$

for $a_j = c_j a$ and $\sum_{j=1}^{N}|c_j|^2 = 1$. Here $2\zeta''/v(\zeta) \equiv \omega''(\zeta)$ and $\omega'(\zeta) + \mathrm{i}\omega''(\zeta) \equiv -\frac{1}{2}\int_{-\infty}^{\infty}\frac{g(\alpha)\,\mathrm{d}\alpha}{\zeta-\alpha}$.

3.3. Soliton collisions

Following Manakov [11], the collision of two vector solitons characterized by the polarizations $\boldsymbol{c}_{(1)}, \boldsymbol{c}_{(2)}$ and by the wavenumbers ζ_1, ζ_2, respectively, results in the change of the polarization of both solitons according to the transformation

$$\begin{aligned} \boldsymbol{c}'_{(1)} &= \frac{1}{\chi}\frac{\zeta_1^* - \zeta_2}{\zeta_1^* - \zeta_2^*}\left[\boldsymbol{c}_{(1)} + \frac{\zeta_2 - \zeta_2^*}{\zeta_2^* - \zeta_1^*}(\boldsymbol{c}_{(2)}^*\cdot\boldsymbol{c}_{(1)})\boldsymbol{c}_{(2)}\right], \\ \boldsymbol{c}'_{(2)} &= \frac{1}{\chi}\frac{\zeta_1^* - \zeta_2}{\zeta_1 - \zeta_2}\left[\boldsymbol{c}_{(2)} + \frac{\zeta_1 - \zeta_1^*}{\zeta_2 - \zeta_1}(\boldsymbol{c}_{(1)}^*\cdot\boldsymbol{c}_{(2)})\boldsymbol{c}_{(1)}\right], \end{aligned} \tag{14}$$

where

$$\chi \equiv \chi(\boldsymbol{c}_{(1)}, \boldsymbol{c}_{(2)}) = \frac{|\zeta_1 - \zeta_2^*|}{|\zeta_1 - \zeta_2|}\left[1 + \frac{(\zeta_1 - \zeta_1^*)(\zeta_2^* - \zeta_2)}{|\zeta_1 - \zeta_2|^2}|\boldsymbol{c}_{(1)}^*\cdot\boldsymbol{c}_{(2)}|^2\right]^{1/2}. \tag{15}$$

The wavenumbers do not change due to the collision. The above transformation is relevant for the collisions of the SF pulses as well as of the SIT pulses, since the inverse scattering equations (of the form of (12)) are similar for both cases (see [6, 11] for comparison).

4. Quantum fluctuations of soliton parameters

Equations of motion for the quantum fluctuations of the soliton parameters are solved for both types of multi-component (SF and SIT) pulses. A transformation of these fluctuations due to the soliton collision is analyzed.

4.1. Fluctuations of SF soliton

Let us evaluate the time dependence of the fluctuations of the soliton parameters $\zeta_j'' \equiv |c_j|\zeta'', \varphi_j, \zeta', x_0$. We use the method introduced in [3] for the scalar SF solitons and developed with relevance to the two-component (Manakov) SF solitons in [25]. The linear (in the soliton-parameter fluctuations) parts of the deviation of the field-operator components from their average values (10) take the form

$$\Delta\epsilon_j(x,\tau) = \epsilon_{j,x_0}(x,\tau)\Delta x_0 + \epsilon_{j,\zeta'}(x,\tau)\Delta\zeta' + \sum_{k=1}^{N}\epsilon_{j,\varphi_k}(x,\tau)\Delta\varphi_k + \sum_{k=1}^{N}\epsilon_{j,\zeta''_k}(x,\tau)\Delta\zeta''_k. \tag{16}$$

Here $\Delta x_0, \Delta\zeta', \Delta\varphi_k, \Delta\zeta''_k$ denote the operators of the fluctuations of the parameters $x_0, \zeta', \varphi_k, \zeta''_k$, respectively. Performing the transformation of field-average derivatives into

$$\begin{cases} \tilde{\epsilon}_{j,x_0} = \epsilon_{j,x_0} \\ \tilde{\epsilon}_{j,\zeta'} = \epsilon_{j,\zeta'} - 2x_0\epsilon_{j,\varphi_j} \\ \tilde{\epsilon}_{j,\varphi_k} = \epsilon_{j,\varphi_k} \\ \tilde{\epsilon}_{j,\zeta''_k} = \epsilon_{j,\zeta''_k}, \end{cases} \tag{17}$$

one finds that $\mathrm{Im}\int_{-\infty}^{\infty}\tilde{\epsilon}^*_{j,A}(x,\tau)\tilde{\epsilon}_{j,B}(x,\tau)\,\mathrm{d}x = 0$ for all (A, B) pairs excluding (x_0, ζ') and (φ_k, ζ''_l) which satisfy

$$\begin{aligned} &\mathrm{Im}\sum_{j=1}^{N}\int_{-\infty}^{\infty}\tilde{\epsilon}^*_{j,x_0}(x,\tau)\tilde{\epsilon}_{j,\zeta'}(x,\tau)\,\mathrm{d}x = 8\zeta'', \\ &\mathrm{Im}\sum_{j=1}^{N}\int_{-\infty}^{\infty}\tilde{\epsilon}^*_{j,\varphi_k}(x,\tau)\tilde{\epsilon}_{j,\zeta''_l}(x,\tau)\,\mathrm{d}x = -16\frac{\zeta''_l}{\zeta''}\delta_{kl} + 4\frac{\zeta''_l\zeta''^2_k}{\zeta''^3}. \end{aligned} \tag{18}$$

Following the above relations, we project the field deviation onto the subspaces relevant to the basis functions $\tilde{\epsilon}_{j,A}(x,\tau)$. For a description shortening, let us define a functional $I[A(x,\tau)] \equiv -\mathrm{i}\int_{-\infty}^{\infty}[A(x,\tau)\Delta\epsilon_j(x,\tau) - \mathrm{h.c.}]\,\mathrm{d}x$ relating to the projections. One finds

$$\begin{aligned} &I[\tilde{\epsilon}^*_{j,x_0}(x,\tau)] = 16\zeta''\Delta\zeta', \\ &I[\tilde{\epsilon}^*_{j,\zeta'}(x,\tau)] = -16\zeta''\Delta x_0, \\ &I[\tilde{\epsilon}^*_{j,\varphi_k}(x,\tau)] = -\frac{1}{\zeta''^3}\left(32\zeta''_k\zeta''^2\Delta\zeta''_k - 8\zeta''^2_k\sum_{l=1}^{N}\zeta''_l\Delta\zeta''_l\right), \\ &I[\tilde{\epsilon}^*_{j,\zeta''_k}(x,\tau)] = \frac{1}{\zeta''^3}\left(32\zeta''_k\zeta''^2\Delta\varphi_k - 8\zeta''_k\sum_{l=1}^{N}\zeta''^2_l\Delta\varphi_l\right) + \frac{1}{\zeta''}48x_0\zeta''_k\Delta\zeta'. \end{aligned} \tag{19}$$

Since the coefficients of the field-deviation expansion in the Heisenberg-picture fluctuation operators differ from those of the above expansion in the Schrödinger-picture operators about a time-dependent phase factor, they are transformed from the Schrödinger picture to the Heisenberg one by taking them at time zero and moving in space, $\tilde{\epsilon}_{j,A}(x,\tau) \to \tilde{\epsilon}_{j,A}(x+4\zeta'\tau, 0)\exp[-\mathrm{i}4(\zeta'^2+\zeta''^2)\tau]$. We determine the time dependence of the soliton-parameter fluctuations as follows:

$$\begin{aligned} &I\left[\tilde{\epsilon}^*_{j,x_0}(x+4\zeta'\tau,0)\,\mathrm{e}^{\mathrm{i}4(\zeta'^2+\zeta''^2)\tau}\right] = 16\zeta''\Delta\zeta'(\tau), \\ &I\left[\tilde{\epsilon}^*_{j,\zeta'}(x+4\zeta'\tau,0)\,\mathrm{e}^{\mathrm{i}4(\zeta'^2+\zeta''^2)\tau}\right] = -16\zeta''\Delta x_0(\tau), \\ &I\left[\tilde{\epsilon}^*_{j,\varphi_k}(x+4\zeta'\tau,0)\,\mathrm{e}^{\mathrm{i}4(\zeta'^2+\zeta''^2)\tau}\right] = -\frac{1}{\zeta''^3}\left[32\zeta''_k\zeta''^2\Delta\zeta''_k(\tau) - 8\zeta''^2_k\sum_{l=1}^{N}\zeta''_l\Delta\zeta''_l(\tau)\right], \\ &I\left[\tilde{\epsilon}^*_{j,\zeta''_k}(x+4\zeta'\tau,0)\,\mathrm{e}^{\mathrm{i}4(\zeta'^2+\zeta''^2)\tau}\right] \\ &\qquad = \frac{1}{\zeta''^3}\left[32\zeta''_k\zeta''^2\Delta\varphi_k(\tau) - 8\zeta''_k\sum_{l=1}^{N}\zeta''^2_l\Delta\varphi_l(\tau)\right] + \frac{1}{\zeta''}48x_0\zeta''_k\Delta\zeta'(\tau). \end{aligned} \tag{20}$$

Evaluating the integrals on the left-hand side of (20), one arrives at the time dependences of the fluctuation operators

$$\begin{aligned}
&\Delta x_0(\tau) = \Delta x_0 - 4\tau\Delta\zeta',\\
&\Delta\zeta'(\tau) = \Delta\zeta',\\
&\Delta\varphi_k(\tau) = \Delta\varphi_k - 8\tau\sum_{l=1}^{N}\zeta_l''\Delta\zeta_l'' \equiv \Delta\varphi_k + \delta\varphi(\tau),\\
&\Delta\zeta_k''(\tau) = \Delta\zeta_k''.
\end{aligned} \tag{21}$$

The characteristic topic of the phase fluctuations $\Delta\varphi_k(\tau)$ is that their time-dependent part $\delta\varphi(\tau)$ is similar for all N modes, which was found earlier for the two-component SF solitons [25].

4.2. Fluctuations of SIT soliton

In order to determine the position dependence of the fluctuations of $\zeta_j'' \equiv |c_j|\zeta''$, φ_j, ζ', τ_0, we adapt the analysis of quantum fluctuations of the SF vector-soliton parameters from subsection 4.1 to the SIT vector solitons following analogous considerations for the scalar SIT solitons [4]. The part of the field-component deviation linear in the fluctuation operators takes the form similar to that of (16)

$$\Delta\epsilon_j(x,\tau) = \epsilon_{j,\tau_0}(x,\tau)\Delta\tau_0 + \epsilon_{j,\zeta'}(x,\tau)\Delta\zeta' + \sum_{k=1}^{N}\epsilon_{j,\varphi_k}(x,\tau)\Delta\varphi_k + \sum_{k=1}^{N}\epsilon_{j,\zeta_k''}(x,\tau)\Delta\zeta_k''. \tag{22}$$

We establish relations between the field-average derivatives over the soliton parameters (the basis functions) transforming them into

$$\begin{cases}
\tilde\epsilon_{j,\tau_0} = \epsilon_{j,\tau_0}\\
\tilde\epsilon_{j,\zeta'} = \epsilon_{j,\zeta'} + (2x/v - x\omega'_{,\zeta'} - 2\tau_0)\epsilon_{j,\varphi_j}\\
\tilde\epsilon_{j,\varphi_k} = \epsilon_{j,\varphi_k}\\
\tilde\epsilon_{j,\zeta_k''} = \epsilon_{j,\zeta_k''} + x(1/v)_{,\zeta''}\dfrac{\zeta_k''}{\zeta''}\epsilon_{j,\tau_0}.
\end{cases} \tag{23}$$

The integrals $\mathrm{Im}\int_{-\infty}^{\infty}\tilde\epsilon_{j,A}^*(x,\tau)\tilde\epsilon_{j,B}(x,\tau)\,\mathrm{d}\tau$ are equal to zero for all the (A,B) pairs except (τ_0,ζ') and (φ_k,ζ_l'') which satisfy

$$\mathrm{Im}\sum_{j=1}^{N}\int_{-\infty}^{\infty}\tilde\epsilon_{j,\tau_0}^*(x,\tau)\tilde\epsilon_{j,\zeta'}(x,\tau)\,\mathrm{d}\tau = 8\zeta'', \tag{24}$$

$$\mathrm{Im}\sum_{j=1}^{N}\int_{-\infty}^{\infty}\tilde\epsilon_{j,\varphi_k}^*(x,\tau)\tilde\epsilon_{j,\zeta_l''}(x,\tau)\,\mathrm{d}\tau = -16\frac{\zeta_l''}{\zeta''}\delta_{kl} + 4\frac{\zeta_l''\zeta_k''^2}{\zeta''^3}. \tag{25}$$

Searching for the position dependence of the soliton-parameter fluctuations, we project the field deviation onto the subspaces relevant to the basis functions $\tilde\epsilon_{j,A}(x,\tau)$. Defining a functional $J[A(x,\tau)] \equiv -\mathrm{i}\int_{-\infty}^{\infty}[A(x,\tau)\Delta\epsilon_j(x,\tau) - \mathrm{h.c.}]\,\mathrm{d}\tau$, we find

$$\begin{aligned}
&J\left[\tilde\epsilon_{j,\tau_0}^*(x,\tau)\right] = 16\zeta''\Delta\zeta',\\
&J\left[\tilde\epsilon_{j,\zeta'}^*(x,\tau)\right] = -16\zeta''\Delta\tau_0 + 16x(1/v)_{,\zeta''}\sum_{l=1}^{N}\zeta_l''\Delta\zeta_l'',
\end{aligned}$$

$$J\left[\tilde{\epsilon}^*_{j,\varphi_k}(x,\tau)\right] = -\frac{1}{\zeta''^3}\left(32\zeta''_k\zeta''^2\Delta\zeta''_k - 8\zeta''^2_k\sum_{l=1}^{N}\zeta''_l\Delta\zeta''_l\right),$$

$$J\left[\tilde{\epsilon}^*_{j,\zeta''_k}(x,\tau)\right] = \frac{1}{\zeta''^3}\left(32\zeta''_k\zeta''^2\Delta\varphi_k - 8\zeta''_k\sum_{l=1}^{N}\zeta''^2_l\Delta\varphi_l\right) + \frac{1}{\zeta''}24(x\omega'_{,\zeta'} - 2x/v + 2\tau_0)\zeta''_k\Delta\zeta'. \tag{26}$$

When the field deviation in Heisenberg-picture-like operators of fluctuations (depending on the space coordinate) is expanded, one takes the coefficients of expansion in the Schrödinger-picture-like operators in the initial position and moved in time. Then, one finds

$$\begin{aligned}
&J\left[\tilde{\epsilon}^*_{j,\tau_0}(0,\tau+x/v)\,\mathrm{e}^{\mathrm{i}(-\omega'+2\zeta'/v)x}\right] = 16\zeta''\Delta\zeta'(x),\\
&J\left[\tilde{\epsilon}^*_{j,\zeta'}(0,\tau+x/v)\,\mathrm{e}^{\mathrm{i}(-\omega'+2\zeta'/v)x}\right] = -16\zeta''\Delta\tau_0(x) + 16x(1/v)_{,\zeta''}\sum_{l=1}^{N}\zeta''_l\Delta\zeta''_l(x),\\
&J\left[\tilde{\epsilon}^*_{j,\varphi_k}(0,\tau+x/v)\,\mathrm{e}^{\mathrm{i}(-\omega'+2\zeta'/v)x}\right] = -\frac{1}{\zeta''^3}\left[32\zeta''_k\zeta''^2\Delta\zeta''_k(x) - 8\zeta''^2_k\sum_{l=1}^{N}\zeta''_l\Delta\zeta''_l(x)\right],\\
&J\left[\tilde{\epsilon}^*_{j,\zeta''_k}(0,\tau+x/v)\,\mathrm{e}^{\mathrm{i}(-\omega'+2\zeta'/v)x}\right] = \frac{1}{\zeta''^3}\left[32\zeta''_k\zeta''^2\Delta\varphi_k(x) - 8\zeta''_k\sum_{l=1}^{N}\zeta''^2_l\Delta\varphi_l(x)\right]\\
&\qquad + \frac{1}{\zeta''}24(x\omega'_{,\zeta'} - 2x/v + 2\tau_0)\zeta''_k\Delta\zeta'(x).
\end{aligned} \tag{27}$$

Evaluating the integrals on the left-hand sides of (27), we arrive at the position dependences of the fluctuations

$$\begin{aligned}
&\Delta\tau_0(x) = \Delta\tau_0 - x(1/v)_{,\zeta'}\Delta\zeta' - \frac{3}{2\zeta''^2}x\omega'_{,\zeta'}\sum_{l=1}^{N}\zeta''_l\Delta\zeta''_l,\\
&\Delta\zeta'(x) = \Delta\zeta',\\
&\Delta\varphi_k(x) = \Delta\varphi_k + \frac{2}{3}x(1/v)_{,\zeta''}\zeta''\Delta\zeta' + \frac{1}{\zeta''}x\omega'_{,\zeta''}\sum_{l=1}^{N}\zeta''_l\Delta\zeta''_l \equiv \Delta\varphi_k + \delta\varphi(x)\\
&\Delta\zeta''_k(x) = \Delta\zeta''_k.
\end{aligned} \tag{28}$$

4.3. Collision-induced fluctuation exchange

From (14) and (15), the polarization parameters $\zeta''_{1k}, \varphi_{1k}$ of one of the colliding solitons transform via the collision into

$$\begin{aligned}
(\zeta''_{1k})' &= (\zeta''_{1k})'(\zeta'_{1j},\zeta'_{2j},\zeta''_{1j},\zeta''_{2j},\varphi_{1j},\varphi_{2j}),\\
(\varphi_{1k})' &= (\varphi_{1k})'(\zeta'_{1j},\zeta'_{2j},\zeta''_{1j},\zeta''_{2j},\varphi_{1j},\varphi_{2j}),
\end{aligned} \tag{29}$$

respectively. Assuming that the collision time is very short compared to the pulse-propagation time, we determine fluctuations of these parameters after the collision $\Delta(\zeta''_{1k})'$, $\Delta(\varphi_{1k})'$ with dependence on the parameter fluctuations before the collision. For our purposes of a long-time processing with a soliton representing a system of cebits, we can neglect the dependence of the polarization on the time-independent (SF) or position-independent (SIT) fluctuations $\Delta\zeta'_{ik}, \Delta\zeta''_{ik}$ $(i = 1, 2)$. Also, we neglect the time (position) dependence of the fluctuations of φ_{2k}, assuming that the soliton indexed by '2' (a switching soliton) propagates through much shorter time than that indexed by '1' (the information register). Thus, in the case of the SF-soliton collision, for a long time of the soliton '1' propagation, such that $|\langle\Delta\varphi_{1k}\rangle| \ll |\langle\delta\varphi_1(\tau)\rangle|$ (where $\delta\varphi_1(\tau)$ is defined by (21)),

$$\begin{aligned}
\Delta(\zeta''_{1k})'\big(\tau^+_{\rm coll}\big) &= \sum_{j=1}^{N} \frac{\partial(\zeta''_{1k})'}{\partial\varphi_{1j}} \Delta\varphi_{1j}\big(\tau^-_{\rm coll}\big) + O(\Delta\zeta'_{ij}, \Delta\zeta''_{ij}, \Delta\varphi_{2j}),\\
\Delta(\varphi_{1k})'\big(\tau^+_{\rm coll}\big) &= \sum_{j=1}^{N} \frac{\partial(\varphi_{1k})'}{\partial\varphi_{1j}} \Delta\varphi_{1j}\big(\tau^-_{\rm coll}\big) + O(\Delta\zeta'_{ij}, \Delta\zeta''_{ij}, \Delta\varphi_{2j}).
\end{aligned} \tag{30}$$

Here $\tau_{\rm coll}$ denotes the moment of the collision. In the case of the SIT-soliton collision, one changes $\tau_{\rm coll}$ in (30) into the position of the collision $x_{\rm coll}$.

5. Collision-based logical gates

We consider a 2^n-component vector soliton of the wavenumber ζ as an n-cebit register. Its polarization (state) vector is transformed after the collision with another soliton (the switching soliton). The parameters of the switching solitons sequences of which correspond to the quantum gates; the polarizations $\boldsymbol{c}_y, \boldsymbol{d}_y, \ldots$, and wavenumbers $\zeta_y, \eta_y, \ldots$ are indexed with $y = a, b, \ldots$ related to a cebit of the register which they switch. We assign consecutive cebits $a, b, \ldots$ to the components $c_1, c_2, \ldots$ of the register polarization following the scheme presented below for a three-cebit case. A state of the system of cebits a, b, c corresponding to the lower, middle and upper wire of quantum circuits respectively is written with the vector

$$\begin{aligned}
|\boldsymbol{c}\rangle = {}& c_1|0\rangle_c|0\rangle_b|0\rangle_a + c_2|0\rangle_c|0\rangle_b|1\rangle_a + c_3|0\rangle_c|1\rangle_b|0\rangle_a + c_4|0\rangle_c|1\rangle_b|1\rangle_a\\
&+ c_5|1\rangle_c|0\rangle_b|0\rangle_a + c_6|1\rangle_c|0\rangle_b|1\rangle_a + c_7|1\rangle_c|1\rangle_b|0\rangle_a + c_8|1\rangle_c|1\rangle_b|1\rangle_a.
\end{aligned} \tag{31}$$

Due to the collision, a state vector of the register $\boldsymbol{c}$ transforms into $L(\boldsymbol{c}_y)\boldsymbol{c}$. Following (14) and (15), $L(\boldsymbol{c}_y)\mathrm{c} \equiv \frac{1}{\chi(\boldsymbol{c}_y,\boldsymbol{c})}\mathcal{L}(\boldsymbol{c}_y)\boldsymbol{c}$, where

$$\mathcal{L}_{ij}(\boldsymbol{c}_y) = \begin{cases} \dfrac{\zeta^* - \zeta_y}{\zeta^* - \zeta^*_y}\left(1 - \dfrac{\zeta_y - \zeta^*_y}{\zeta^* - \zeta^*_y} c_{yi} c^*_{yj}\right) & i = j\\[2ex] -\dfrac{\zeta^* - \zeta_y}{\zeta^* - \zeta^*_y}\dfrac{\zeta_y - \zeta^*_y}{\zeta^* - \zeta^*_y} c_{yi} c^*_{yj} & i \neq j \end{cases} \tag{32}$$

and $\chi(\boldsymbol{c}_y, \boldsymbol{c}) = |\mathcal{L}(\boldsymbol{c}_y)\boldsymbol{c}|$. Since all the logical gates relate to linear transformations of the state vector while $L(\boldsymbol{c}_y)$ is nonlinear in general, we consider such $\boldsymbol{c}_y$ that $\mathcal{L}(\boldsymbol{c}_y)$ is unitary, thus $L(\boldsymbol{c}_y) = \mathcal{L}(\boldsymbol{c}_y)$ and $L(\boldsymbol{c}_y)$ is linear.

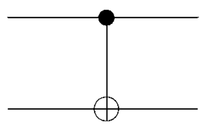
In [2], the parameters of the switching solitons changing the register-soliton state vector via the collisions as the quantum gates of the universal set [7]; *CNOT*, *Hadamard*, $\pi/8$, *phase* have been found. Here, I present the *CNOT*-gate realization in detail and outline the *Hadamard* and $\pi/8$-gate realizations. The gate *phase* is known to be the composition of two $\pi/8$ gates. Let us mention that the linearization of the $\boldsymbol{c}$-vector transform (32) is approximate in the case of the *CNOT* and *Hadamard* operations (as described in the following paragraph). I refer to the resulting error at the end of section 6.

The *CNOT* operation consists of two steps; rotating the polarization vector of the register and multiplying the resulting polarization by '-1'. Here, I present the first step addressing the second step to [2]. The rotation demands satisfaction of the condition $\zeta'' \ll \zeta''_y$ together with: (i) $\zeta' = \zeta'_y$ or (ii) $|\zeta' - \zeta'_y| \ll \zeta''_y$. Following the first condition, our implementation of the *CNOT* gate is approximate. The computation accuracy depends on the ratio of the register and switching pulse widths, thus it is determined by the method of creating the pulses. However, use of the condition (ii) instead of (i) results in further loss of the computation accuracy. The condition (i) can be fulfilled for collisions of the SIT pulses while it is inapplicable to SF pulses since values $4\zeta'$, $4\zeta'_y$ are equal to the SF-soliton velocities following (10). Thus, if $\zeta' = \zeta'_y$, the relevant SF solitons cannot collide. Processing with two-cebit information, we take the components of the polarization-vector of the switching soliton $c_{yi} \equiv |c_{yi}|\,\mathrm{e}^{\mathrm{i}\varphi_{yi}}$ for $y = a$ such that

$$c_{a1} = c_{a2} = 0, \qquad |c_{a3}| = |c_{a4}| = \tfrac{1}{\sqrt{2}}, \qquad \varphi_{a3} - \varphi_{a4} = (2k+1)\pi, \tag{33}$$

where k is an integer and we find

$$L(\boldsymbol{c}_a) \approx (-1)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \tag{34}$$

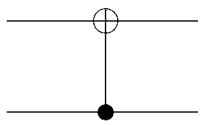


Up to the multiplier '-1', $L(\boldsymbol{c}_a)$ is the *CNOT* operator represented graphically as the quantum circuit above. The *CNOT* operation changing the cebit b can be performed with the switching soliton of the polarization $\boldsymbol{c}_b$

$$c_{b1} = c_{b3} = 0, \qquad |c_{b2}| = |c_{b4}| = \tfrac{1}{\sqrt{2}}, \qquad \varphi_{b2} - \varphi_{b4} = (2k+1)\pi, \tag{35}$$

where k denotes an integer. The relevant transformation matrix of the register polarization is

$$L(\boldsymbol{c}_b) \approx (-1)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{36}$$

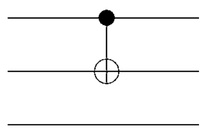


Performing a gate operation on the n-cebit state demands use of 2^{n-2}-times more soliton collisions than for two cebits. Let us study the implementation of the *CNOT* operation increasing the number of cebits. We begin by adding a 'free' wire (a cebit unchanged via the gate action) to the bottom of the circuit (34). The gate is realized via two collisions of the eight-component register. The switching solitons of the polarizations $\boldsymbol{c}_b$, $\boldsymbol{d}_b$ such that

$$\begin{aligned} &|c_{b6}| = |c_{b8}| = |d_{b5}| = |d_{b7}| = 1/\sqrt{2}, \\ &\varphi_{b6} - \varphi_{b8} = (2k+1)\pi, \qquad \phi_{b5} - \phi_{b7} = (2l+1)\pi \end{aligned} \tag{37}$$

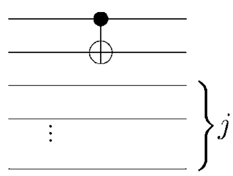
transform the register following the matrix

$$[\mathrm{CNOT}]_1 \equiv L(\boldsymbol{d}_b)L(\boldsymbol{c}_b) = \begin{pmatrix} 1_2 & 0 & 0 & 0 \\ 0 & 1_2 & 0 & 0 \\ 0 & 0 & 0 & 1_2 \\ 0 & 0 & 1_2 & 0 \end{pmatrix}, \tag{38}$$



where 1_n denotes the unit matrix of rank n. The addition of j free wires to the bottom of the circuit (34) corresponds to the *CNOT* transformation represented as

$$[\mathrm{CNOT}]_j = \begin{pmatrix} 1_{2^j} & 0 & 0 & 0 \\ 0 & 1_{2^j} & 0 & 0 \\ 0 & 0 & 0 & 1_{2^j} \\ 0 & 0 & 1_{2^j} & 0 \end{pmatrix}, \tag{39}$$



which can be performed with 2^j collisions of the register with the solitons of the linearly independent polarizations

$$\begin{aligned}
\boldsymbol{c}_y &= \Big[\underbrace{0,\ldots,0}_{2^{j+2}-2^j-1},\ \tfrac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\varphi_y},\ \underbrace{0,\ldots,0}_{2^j-1},\ -\tfrac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\varphi_y}\Big],\\
\boldsymbol{d}_y &= \Big[\underbrace{0,\ldots,0}_{2^{j+2}-2^j-2},\ \tfrac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\phi_y},\ \underbrace{0,\ldots,0}_{2^j-1},\ -\tfrac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\phi_y},\ 0\Big],\\
\boldsymbol{e}_y &= \Big[\underbrace{0,\ldots,0}_{2^{j+2}-2^j-3},\ \tfrac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\psi_y},\ \underbrace{0,\ldots,0}_{2^j-1},\ -\tfrac{1}{\sqrt{2}}\,\mathrm{e}^{\mathrm{i}\psi_y},\ 0,\ 0\Big],\\
&\ \ \vdots
\end{aligned} \tag{40}$$

In the next step, we add m free wires to the top of the circuit. The relevant transformation matrix is block diagonal with 2^m identical $[\mathrm{CNOT}]_j$ matrices on the diagonal. It can be decomposed into the product of 2^m block-diagonal matrices containing $2^m - 1$ unit blocks and one $[\mathrm{CNOT}]_j$ block. All of the operations corresponding to the factor matrices can be performed independently with 2^j collisions since the polarizations of their switching solitons are linearly independent. The composition of these operations demands $2^{j+m} = 2^{n-2}$ collisions.

The *Hadamard* operation is performed with similar assumptions on the real and imaginary parts of the switching-soliton wavenumbers to those for the *CNOT* operation. One switches the one-cebit register with the polarization of the switching soliton

$$\boldsymbol{c}_a = \left(\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}\,\mathrm{e}^{\mathrm{i}\varphi_a},\ -\sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}\,\mathrm{e}^{\mathrm{i}\varphi_a}\right), \tag{41}$$

where φ_a is an arbitrary real number. In order to perform the $\pi/8$ operation on the one-cebit register, one collides it with a soliton satisfying the condition for the wavenumbers

$$\frac{\zeta^* - \zeta_a}{\zeta^* - \zeta_a^*} = \mathrm{e}^{\mathrm{i}\pi/4}, \tag{42}$$

and of the polarization $\boldsymbol{c}_a = (0, 1)$. The generalization of the one-cebit operations to the case when the register consists of n cebits demands compositions of 2^{n-1} consecutive switching solitons and it follows the above considerations for the *CNOT* gate [2].

6. Errors and their correction

Errors of the present method of information processing result from the fluctuation of their parameters and from the exchange of these fluctuations during the collision described in section 4. Below, I show that the most important error to be corrected results from time dependent propagational fluctuations of the phase factors $\Delta\varphi_k(\tau)$ and that is not significantly influenced by the soliton collisions.

Let us investigate the collision-induced exchange of the parameter fluctuations. We note that all the switching solitons are pulses of one non-zero polarization component ($\pi/8$, *phase* gates, the operation of the state-vector multiplication by a number) or of two non-zero components (*CNOT*, *Hadamard* gates). Following (29), let $\zeta_k''\,\mathrm{e}^{\mathrm{i}\varphi_k}$, $(\zeta_k'')'\,\mathrm{e}^{\mathrm{i}(\varphi_k)'}$ denote the polarization components of the register before and after the collision, respectively. The one-component switching solitons do not influence $(\zeta_k'')'$ while the collisions with the switching solitons of the *CNOT* gate result in exchanging two of the register-polarization components $\zeta_k''\,\mathrm{e}^{\mathrm{i}\varphi_k}$. Thus, for all these collisions, $\partial(\zeta_k'')'/\partial\varphi_j = 0$ and $\partial(\varphi_k)'/\partial\varphi_j = 0$ or $\partial(\varphi_k)'/\partial\varphi_j = 1$.

The switching solitons of the *Hadamard* gate influence two of the register-polarization components, say j th and k th components, as follows:

$$(\zeta_j'')' \, \mathrm{e}^{\mathrm{i}(\varphi_j)'} = \tfrac{1}{\sqrt{2}}\big(\zeta_j'' \, \mathrm{e}^{\mathrm{i}\varphi_j} + \zeta_k'' \, \mathrm{e}^{\mathrm{i}\varphi_k}\big), \qquad (\zeta_k'')' \, \mathrm{e}^{\mathrm{i}(\varphi_k)'} = \tfrac{1}{\sqrt{2}}\big(\zeta_j'' \, \mathrm{e}^{\mathrm{i}\varphi_j} - \zeta_k'' \, \mathrm{e}^{\mathrm{i}\varphi_k}\big), \tag{43}$$

leaving unchanged other polarization components. Using (30), (43), one finds the propagation-time dependent parts of the fluctuations $\Delta(\zeta_k'')'$ to be equal to zero

$$\begin{aligned}\Delta(\zeta_{k(j)}'')'\big(\tau_{\mathrm{coll}}^{+}\big) &= \delta\varphi\big(\tau_{\mathrm{coll}}^{-}\big)\left[\frac{\partial(\zeta_{k(j)}'')'}{\partial\varphi_k} + \frac{\partial(\zeta_{k(j)}'')'}{\partial\varphi_j}\right] + O(\Delta\zeta_{lm}', \Delta\zeta_{lm}'', \Delta\varphi_{lm}) \\ &= O(\Delta\zeta_{lm}', \Delta\zeta_{lm}'', \Delta\varphi_{lm})\end{aligned} \tag{44}$$

and

$$\begin{aligned}\Delta(\varphi_{k(j)})'\big(\tau_{\mathrm{coll}}^{+}\big) &= \delta\varphi\big(\tau_{\mathrm{coll}}^{-}\big)\left[\frac{\partial(\varphi_{k(j)})'}{\partial\varphi_k} + \frac{\partial(\varphi_{k(j)})'}{\partial\varphi_j}\right] + O(\Delta\zeta_{lm}', \Delta\zeta_{lm}'', \Delta\varphi_{lm}) \\ &= \delta\varphi\big(\tau_{\mathrm{coll}}^{-}\big) + O(\Delta\zeta_{lm}', \Delta\zeta_{lm}'', \Delta\varphi_{lm}).\end{aligned} \tag{45}$$

Thus, for all the soliton collisions relevant to the logical gates, we have shown that the fluctuations $\Delta(\zeta_k'')'$ are independent of $\delta\varphi(\tau)$ and

$$\Delta(\varphi_k)'\big(\tau_{\mathrm{coll}}^{+}\big) \approx \delta\varphi\big(\tau_{\mathrm{coll}}^{-}\big). \tag{46}$$

This result indicates that the collisions of the long-time propagated register soliton do not influence significantly its polarization fluctuations. The time dependences of the register-polarization fluctuations are similar to those of the soliton propagated without collisions. This property is irrelevant for soliton collisions in general, in particular, when the polarization transform (14) is non-reducible to (32).

In order to establish the maximal time length of the register-soliton propagation (the maximal time of the information processing), we ask which values of $\langle\Delta\varphi_k(\tau)\rangle$ are acceptable for performing logical operations. Let us note that every soliton collision relates to an unitary transformation in a two-dimensional subspace of the polarization-vector space. Such transformations can be decomposed into rotations about Euler angles up to an additional phase factor following $U(\alpha,\beta,\gamma,\delta) = \mathrm{e}^{\mathrm{i}\alpha} R_z(\beta) R_y(\gamma) R_z(\delta)$, where

$$R_y(\theta) = \begin{pmatrix}\cos(\theta/2) & -\sin(\theta/2)\\ \sin(\theta/2) & \cos(\theta/2)\end{pmatrix}, \qquad R_z(\theta) = \begin{pmatrix}\mathrm{e}^{-\mathrm{i}\theta/2} & 0\\ 0 & \mathrm{e}^{\mathrm{i}\theta/2}\end{pmatrix}. \tag{47}$$

In particular, the collisions composing into *Hadamard* and $\pi/8$ operations correspond to the transformations $[H] = U(\pi/2, 0, \pi/2, \pi)$, $[\pi/8] = U(\pi/8, 0, 0, \pi/4)$, respectively, while the collisions of the *CNOT* gate are represented by

$$[\mathrm{CNOT}] = \begin{pmatrix}1_2 & 0\\ 0 & U(\pi/2, 0, \pi, \pi)\end{pmatrix}. \tag{48}$$

The elementary operations of the logical gates (collisions) are efficient as long as $\alpha, \gamma, \delta \gg |\langle\Delta\varphi_k(\tau_{\mathrm{coll}})\rangle|$, which leads to the condition $|\langle\Delta\varphi_k(\tau_{\mathrm{coll}})\rangle| \ll \pi/8$. Thus, any logical algorithm can be efficiently performed for τ_{coll} small enough to satisfy the last relation, while the value of the phase fluctuation in this inequality $\langle\Delta\varphi_k(\tau_{\mathrm{coll}})\rangle$ depends on initial conditions for the soliton-parameter fluctuations.

As was mentioned in the introduction, the lack of nonlocality of many-cebit information leads to inefficiency of usual quantum error-correcting networks. One needs another method of the error minimization than those correcting bit-flip or phase-flip errors. An efficient way is repeating the algorithm and averaging the results of the state-parameter measurements. It is possible because of specific properties of the switching solitons relevant to the universal set of operations. In particular, the switching solitons realizing the *CNOT* and *Hadamard* gates are of the property $\zeta'' \ll \zeta_y''$, where ζ'' and ζ_y'' denote the wavenumber imaginary parts of the register and switching solitons respectively. Following (14) and (15), with this condition, the switching-soliton polarization remains unchanged after the collision with the consecutive register solitons. The same is true for the $\pi/8$ gate if the additional to (42) condition $\zeta'' \ll \zeta_y''$ is satisfied. If a number of identically prepared register solitons were propagated through the same circuit (collided with a sequence of the same switching pulses), their polarization would be changed in the same way. When measuring the final polarizations of all the register pulses and averaging them, the polarization fluctuations connected to different pulses cancel each other.

For the case of the presence of many solitons in the system at the same time, in particular, many registers, we assume that their interactions are pairwise (the pulse-polarization transformation of a soliton colliding with a sequence of other solitons factorizes into a product of transformations (14)) following [11]. This factorization has been proved recently to be accurate [26], thus, it is not connected to an additional error.

Our error analysis was incomplete without discussing consequences of the assumption on the big soliton-width difference $\zeta'' \ll \zeta_y''$ that was important for the linearization of the polarization transform (32) relevant to *CNOT* or *Hadamard* operations. In order to precise this condition, we note that the relative error of the real and imaginary parts of the register-polarization components induced by the *CNOT* or *Hadamard* operations is of the order of ζ''/ζ_y''. Since this error grows linearly with the operation number, the pulse-amplitude ratio ζ''/ζ_y'' should be much smaller than the inverse of the number of the *CNOT* and *Hadamard* operations in the algorithm. The same pulse-amplitude ratio is important for the efficiency of the error-minimization method proposed above. In order to avoid cumulative influence of the collisions with consecutive register pulses on the switching solitons, ζ''/ζ_y'' should be much smaller then the inverse of the register number. This condition is, however, satisfied when there are more logical operations than the error-minimizing algorithm repetitions.

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